Prerequisite Skills

Operations with Fractions: Adding and Subtracting

- To add or subtract fractions with the same denominator, add or subtract the numerators and write the sum or difference over the denominator.

Example 1 Find each sum or difference.

a. $\frac{3}{5} + \frac{1}{5}$

$$\frac{3}{5} + \frac{1}{5} = \frac{3+1}{5} \quad \text{The denominators are the same. Add the numerators.}$$

$$= \frac{4}{5} \quad \text{Simplify.}$$

b. $\frac{5}{9} - \frac{4}{9}$

$$\frac{5}{9} - \frac{4}{9} = \frac{5-4}{9} \quad \text{The denominators are the same. Subtract the numerators.}$$

$$= \frac{1}{9} \quad \text{Simplify.}$$

- To write a fraction in simplest form, divide both the numerator and the denominator by their greatest common factor (GCF).

Example 2 Write each fraction in simplest form.

a. $\frac{4}{16}$

$$\frac{4}{16} = \frac{4 \div 4}{16 \div 4} \quad \text{Divide 4 and 16 by their GCF, 4.}$$

$$= \frac{1}{4} \quad \text{Simplify.}$$

b. $\frac{24}{36}$

$$\frac{24}{36} = \frac{24 \div 12}{36 \div 12} \quad \text{Divide 24 and 36 by their GCF, 12.}$$

$$= \frac{2}{3} \quad \text{Simplify.}$$

Example 3 Find each sum or difference. Write in simplest form.

a. $\frac{7}{16} - \frac{1}{16}$

$$\frac{7}{16} - \frac{1}{16} = \frac{6}{16} \quad \text{The denominators are the same. Subtract the numerators.}$$

$$= \frac{3}{8} \quad \text{Simplify.}$$

b. $\frac{5}{8} + \frac{7}{8}$

$$\frac{5}{8} + \frac{7}{8} = \frac{12}{8} \quad \text{The denominators are the same. Add the numerators.}$$

$$= 1\frac{4}{8} \text{ or } 1\frac{1}{2} \quad \text{Rename } \frac{12}{8} \text{ as a mixed number in simplest form.}$$

- To add or subtract fractions with unlike denominators, first find the least common denominator (LCD). Rename each fraction with the LCD, and then add or subtract. Simplify if necessary.

Example **4**

Find each sum or difference. Write in simplest form.

a. $\frac{2}{9} + \frac{1}{3}$

$$\frac{2}{9} + \frac{1}{3} = \frac{2}{9} + \frac{3}{9}$$ The LCD for 9 and 3 is 9. Rename $\frac{1}{3}$ as $\frac{3}{9}$.

$$= \frac{5}{9}$$ Add the numerators.

b. $\frac{1}{2} + \frac{2}{3}$

$$\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6}$$ The LCD for 2 and 3 is 6. Rename $\frac{1}{2}$ as $\frac{3}{6}$ and $\frac{2}{3}$ as $\frac{4}{6}$.

$$= \frac{7}{6} \text{ or } 1\frac{1}{6}$$ Simplify.

c. $\frac{3}{8} - \frac{1}{3}$

$$\frac{3}{8} - \frac{1}{3} = \frac{9}{24} - \frac{8}{24}$$ The LCD for 8 and 3 is 24. Rename $\frac{3}{8}$ as $\frac{9}{24}$ and $\frac{1}{3}$ as $\frac{8}{24}$.

$$= \frac{1}{24}$$ Simplify.

d. $\frac{7}{10} - \frac{2}{15}$

$$\frac{7}{10} - \frac{2}{15} = \frac{21}{30} - \frac{4}{30}$$ The LCD for 10 and 15 is 30. Rename $\frac{7}{10}$ as $\frac{21}{30}$ and $\frac{2}{15}$ as $\frac{4}{30}$.

$$= \frac{17}{30}$$ Simplify.

Exercises   **Find each sum or difference.**

1. $\frac{2}{5} + \frac{1}{5}$ **2.** $\frac{2}{7} - \frac{1}{7}$ **3.** $\frac{4}{3} + \frac{4}{3}$

4. $\frac{3}{9} + \frac{4}{9}$ **5.** $\frac{5}{16} - \frac{4}{16}$ **6.** $\frac{7}{2} - \frac{4}{2}$

Simplify.

7. $\frac{6}{9}$ **8.** $\frac{7}{14}$ **9.** $\frac{28}{40}$

10. $\frac{16}{100}$ **11.** $\frac{27}{99}$ **12.** $\frac{24}{180}$

Find each sum or difference. Write in simplest form.

13. $\frac{2}{9} + \frac{1}{9}$ **14.** $\frac{2}{15} + \frac{7}{15}$ **15.** $\frac{2}{3} + \frac{1}{3}$

16. $\frac{7}{8} - \frac{3}{8}$ **17.** $\frac{4}{9} - \frac{1}{9}$ **18.** $\frac{5}{4} - \frac{3}{4}$

19. $\frac{1}{2} + \frac{1}{4}$ **20.** $\frac{1}{2} - \frac{1}{3}$ **21.** $\frac{4}{3} + \frac{5}{9}$

22. $1\frac{1}{2} - \frac{3}{2}$ **23.** $\frac{1}{4} + \frac{1}{5}$ **24.** $\frac{2}{3} + \frac{1}{4}$

25. $\frac{3}{2} + \frac{1}{2}$ **26.** $\frac{8}{9} - \frac{2}{3}$ **27.** $\frac{3}{7} + \frac{5}{14}$

28. $\frac{13}{20} - \frac{2}{5}$ **29.** $1 - \frac{1}{19}$ **30.** $\frac{9}{10} - \frac{3}{5}$

31. $\frac{3}{4} - \frac{2}{3}$ **32.** $\frac{4}{15} + \frac{3}{4}$ **33.** $\frac{11}{12} - \frac{4}{15}$

34. $\frac{3}{11} + \frac{1}{8}$ **35.** $\frac{94}{100} - \frac{11}{25}$ **36.** $\frac{3}{25} + \frac{5}{6}$

Operations with Fractions: Multiplying and Dividing

• To multiply fractions, multiply the numerators and multiply the denominators.

Example 1 **Find each product.**

a. $\dfrac{2}{5} \cdot \dfrac{1}{3}$

$$\dfrac{2}{5} \cdot \dfrac{1}{3} = \dfrac{2 \cdot 1}{5 \cdot 3}$$ Multiply the numerators.
Multiply the denominators.

$$= \dfrac{2}{15}$$ Simplify.

b. $\dfrac{7}{3} \cdot \dfrac{1}{11}$

$$\dfrac{7}{3} \cdot \dfrac{1}{11} = \dfrac{7 \cdot 1}{3 \cdot 11}$$ Multiply the numerators.
Multiply the denominators.

$$= \dfrac{7}{33}$$ Simplify.

• If the fractions have common factors in the numerators and denominators, you can simplify before you multiply by canceling.

Example 2 **Find each product. Simplify before multiplying.**

a. $\dfrac{3}{4} \cdot \dfrac{4}{7}$

$$\dfrac{3}{4} \cdot \dfrac{4}{7} = \dfrac{3}{\overset{}{\underset{1}{4}}} \cdot \dfrac{\overset{1}{4}}{7}$$ Divide by the GCF, 4.

$$= \dfrac{3}{7}$$ Simplify.

b. $\dfrac{4}{9} \cdot \dfrac{45}{49}$

$$\dfrac{4}{9} \cdot \dfrac{45}{49} = \dfrac{4}{\overset{}{\underset{1}{9}}} \cdot \dfrac{\overset{5}{45}}{49}$$ Divide by the GCF, 9.

$$= \dfrac{20}{49}$$ Multiply the numerators and denominators.

• Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**.

Example 3 **Name the reciprocal of each number.**

a. $\dfrac{3}{8}$

$\dfrac{3}{8} \cdot \dfrac{8}{3} = 1$ The product is 1.

The reciprocal of $\dfrac{3}{8}$ is $\dfrac{8}{3}$.

b. $\dfrac{1}{6}$

$\dfrac{1}{6} \cdot \dfrac{6}{1} = 1$ The product is 1.

The reciprocal of $\dfrac{1}{6}$ is 6.

c. $2\dfrac{4}{5}$

$2\dfrac{4}{5} = \dfrac{14}{5}$ Write $2\dfrac{4}{5}$ as an improper fraction.

$\dfrac{14}{5} \cdot \dfrac{5}{14} = 1$ The product is 1.

The reciprocal of $2\dfrac{4}{5}$ is $\dfrac{5}{14}$.

CONTENTS

- To divide one fraction by another fraction, multiply the dividend by the multiplicative inverse of the divisor.

Example 4 Find each quotient.

a. $\dfrac{1}{3} \div \dfrac{1}{2}$

$\dfrac{1}{3} \div \dfrac{1}{2} = \dfrac{1}{3} \cdot \dfrac{2}{1}$ Multiply $\dfrac{1}{3}$ by $\dfrac{2}{1}$, the reciprocal of $\dfrac{1}{2}$.

$= \dfrac{2}{3}$ Simplify.

b. $\dfrac{3}{8} \div \dfrac{2}{3}$

$\dfrac{3}{8} \div \dfrac{2}{3} = \dfrac{3}{8} \cdot \dfrac{3}{2}$ Multiply $\dfrac{3}{8}$ by $\dfrac{3}{2}$, the reciprocal of $\dfrac{2}{3}$.

$= \dfrac{9}{16}$ Simplify.

c. $4 \div \dfrac{5}{6}$

$4 \div \dfrac{5}{6} = \dfrac{4}{1} \cdot \dfrac{6}{5}$ Multiply 4 by $\dfrac{6}{5}$, the reciprocal of $\dfrac{5}{6}$.

$= \dfrac{24}{5}$ or $4\dfrac{4}{5}$ Simplify.

d. $\dfrac{3}{4} \div 2\dfrac{1}{2}$

$\dfrac{3}{4} \div 2\dfrac{1}{2} = \dfrac{3}{4} \cdot \dfrac{2}{5}$ Multiply $\dfrac{3}{4}$ by $\dfrac{2}{5}$, the reciprocal of $2\dfrac{1}{2}$.

$= \dfrac{6}{20}$ or $\dfrac{3}{10}$ Simplify.

Exercises Find each product.

1. $\dfrac{3}{4} \cdot \dfrac{1}{5}$
2. $\dfrac{2}{7} \cdot \dfrac{1}{3}$
3. $\dfrac{1}{5} \cdot \dfrac{3}{20}$
4. $\dfrac{2}{5} \cdot \dfrac{3}{7}$

5. $\dfrac{5}{2} \cdot \dfrac{1}{4}$
6. $\dfrac{7}{2} \cdot \dfrac{3}{2}$
7. $\dfrac{1}{3} \cdot \dfrac{2}{5}$
8. $\dfrac{2}{3} \cdot \dfrac{1}{11}$

Find each product. Simplify before multiplying if possible.

9. $\dfrac{2}{9} \cdot \dfrac{1}{2}$
10. $\dfrac{15}{2} \cdot \dfrac{7}{15}$
11. $\dfrac{3}{2} \cdot \dfrac{1}{3}$
12. $\dfrac{1}{3} \cdot \dfrac{6}{5}$

13. $\dfrac{9}{4} \cdot \dfrac{1}{18}$
14. $\dfrac{11}{3} \cdot \dfrac{9}{44}$
15. $\dfrac{2}{7} \cdot \dfrac{14}{3}$
16. $\dfrac{2}{11} \cdot \dfrac{110}{17}$

17. $\dfrac{1}{3} \cdot \dfrac{12}{19}$
18. $\dfrac{1}{3} \cdot \dfrac{15}{2}$
19. $\dfrac{30}{11} \cdot \dfrac{1}{3}$
20. $\dfrac{6}{5} \cdot \dfrac{10}{12}$

Name the reciprocal of each number.

21. $\dfrac{6}{7}$
22. $\dfrac{3}{2}$
23. $\dfrac{1}{22}$
24. $\dfrac{14}{23}$
25. $2\dfrac{3}{4}$
26. $5\dfrac{1}{3}$

Find each quotient.

27. $\dfrac{2}{3} \div \dfrac{1}{3}$
28. $\dfrac{16}{9} \div \dfrac{4}{9}$
29. $\dfrac{3}{2} \div \dfrac{1}{2}$
30. $\dfrac{3}{7} \div \dfrac{1}{5}$

31. $\dfrac{9}{10} \div \dfrac{3}{7}$
32. $\dfrac{1}{2} \div \dfrac{3}{5}$
33. $2\dfrac{1}{4} \div \dfrac{1}{2}$
34. $1\dfrac{1}{3} \div \dfrac{2}{3}$

35. $\dfrac{11}{12} \div 1\dfrac{2}{3}$
36. $\dfrac{3}{8} \div \dfrac{1}{4}$
37. $\dfrac{1}{3} \div 1\dfrac{1}{5}$
38. $\dfrac{3}{25} \div \dfrac{2}{15}$

CONTENTS

The Percent Proportion

- A **percent** is a ratio that compares a number to 100. To write a percent as a fraction, express the ratio as a fraction with a denominator of 100. Fractions should be stated in simplest form.

Example 1 **Express each percent as a fraction.**

a. **25%**

$$25\% = \frac{25}{100} \text{ or } \frac{1}{4} \qquad \text{Definition of percent}$$

b. **107%**

$$107\% = \frac{107}{100} \text{ or } 1\frac{7}{100} \qquad \text{Definition of percent}$$

c. **0.5%**

$$0.5\% = \frac{0.5}{100} \qquad \text{Definition of percent}$$

$$= \frac{5}{1000} \text{ or } \frac{1}{200} \qquad \text{Simplify.}$$

- In the **percent proportion**, the ratio of a part of something (part) to the whole (base) is equal to the percent written as a fraction.

part → $\frac{a}{b} = \frac{p}{100}$ ← percent

base

Example: $\underbrace{10}_{\text{part}}$ is $\underbrace{25\%}_{\text{percent}}$ of $\underbrace{40}_{\text{base}}$.

Example 2 **40% of 30 is what number?**

The percent is 40, and the base is 30. Let a represent the part.

$$\frac{a}{b} = \frac{p}{100} \qquad \text{Use the percent proportion}$$

$$\frac{a}{30} = \frac{40}{100} \qquad \text{Replace } b \text{ with 30 and } p \text{ with 40.}$$

$$100a = 30(40) \qquad \text{Find the cross products.}$$

$$100a = 1200 \qquad \text{Simplify.}$$

$$\frac{100a}{100} = \frac{1200}{100} \qquad \text{Divide each side by 100.}$$

$$a = 12 \qquad \text{Simplify.}$$

The part is 12. So, 40% of 30 is 12.

Example 3 **Kelsey took a survey of some of the students in her lunch period. 42 out of the 70 students Kelsey surveyed said their family had a pet. What percent of the students had pets?**

You know the part, 42, and the base, 70.
Let p represent the percent.

$$\frac{a}{b} = \frac{p}{100} \qquad \text{Use the percent proportion.}$$

$$\frac{42}{70} = \frac{p}{100} \qquad \text{Replace } a \text{ with 42 and } b \text{ with 70.}$$

$$4200 = 70p \qquad \text{Find the cross products.}$$

$$\frac{4200}{70} = \frac{70p}{70} \qquad \text{Divide each side by 70.}$$

$$60 = p \qquad \text{Simplify.}$$

The percent is 60, so $\frac{60}{100}$ or 60% of the students had pets.

CONTENTS

Example **4**   **67.5 is 75% of what number?**

You know the percent, 75, and the part, 67.5.
Let b represent the base.

$\dfrac{a}{b} = \dfrac{p}{100}$ Use the percent proportion.

$\dfrac{67.5}{b} = \dfrac{75}{100}$ $75\% = \dfrac{75}{100}$, so $p = 75$.
   Replace a with 67.5 and p with 75.

$6750 = 75b$ Find the cross products.

$\dfrac{6750}{75} = \dfrac{75b}{75}$ Divide each side by 75.

$90 = b$ Simplify.

The base is 90, so 67.5 is 75% of 90.

Exercises   **Express each percent as a fraction.**

1. 5%   **2.** 60%   **3.** 11%

4. 120%   **5.** 78%   **6.** 2.5%

7. 0.9%   **8.** 0.4%   **9.** 1400%

Use the percent proportion to find each number.

10. 25 is what percent of 125?   **11.** 16 is what percent of 40?

12. 14 is 20% of what number?   **13.** 50% of what number is 80?

14. What number is 25% of 18?   **15.** Find 10% of 95.

16. What percent of 48 is 30?   **17.** What number is 150% of 32?

18. 5% of what number is 3.5?   **19.** 1 is what percent of 400?

20. Find 0.5% of 250.   **21.** 49 is 200% of what number?

22. 15 is what percent of 12?   **23.** 48 is what percent of 32?

24. Madeline usually makes 85% of her shots in basketball. If she shoots 20 shots, how many will she likely make?

25. Brian answered 36 items correctly on a 40-item test. What percent did he answer correctly?

26. José told his dad that he won 80% of the solitaire games he played yesterday. If he won 4 games, how many games did he play?

27. A glucose solution is prepared by dissolving 6 grams of glucose in 120 milliliters of solution. What is the percent of glucose in the solution?

HEALTH   **For Exercises 28–30, use the following information.**
The U.S. Food and Drug Administration requires food manufacturers to label their products with a nutritional label. The sample label shown at the right shows a portion of the information from a package of macaroni and cheese.

28. The label states that a seving contains 3 grams of saturated fat, which is 15% of the daily value recommended for a 2000-Calorie diet. How many grams of saturated fat are recommended for a 2000-Calorie diet.

29. The 470 milligrams of sodium (salt) in the macaroni and cheese is 20% of the recommended daily value. What is the recommended daily value of sodium?

30. For a healthy diet, the National Research Council recommends that no more than 30 percent of total Calories come from fat. What percent of the Calories in a serving of this macaroni and cheese come from fat?

Nutrition Facts		
Serving Size 1 cup (228g)		
Servings per container 2		
Amount per serving		
Calories 250 Calories from Fat 110		
		%Daily value*
Total Fat 12g		**18%**
Saturated Fat 3g		**15%**
Cholesterol 30mg		**10%**
Sodium 470mg		**20%**
Total Carbohydrate 31g		**10%**
Dietary Fiber 0g		**0%**
Sugars 5g		
Protein 5g		
Vitamin A 4%	•	Vitamin C 2%
Calcium 20%	•	Iron 4%

Expressing Fractions as Decimals and Percents

- To write a fraction as a decimal, divide the numerator by the denominator. To write a decimal as a fraction, write the decimal as a fraction with denominator of 10, 100, 1000, … . Then simplify if possible.

Example 1   **Write each fraction as a decimal.**

a. $\dfrac{5}{8}$

$\dfrac{5}{8} = 5 \div 8$

$\quad = 0.625$

b. $\dfrac{3}{5}$

$\dfrac{3}{5} = 3 \div 5$

$\quad = 0.6$

c. $\dfrac{1}{3}$

$\dfrac{1}{3} = 1 \div 3$

$\quad = 0.333\ldots$

Example 2   **Write each decimal as a fraction.**

a. **0.4**

$0.4 = \dfrac{4}{10}$ or $\dfrac{2}{5}$

b. **0.005**

$0.005 = \dfrac{5}{1000}$ or $\dfrac{1}{200}$

c. **0.98**

$0.98 = \dfrac{98}{100}$ or $\dfrac{49}{50}$

- To write a fraction for a repeating decimal, use the method in Example 3 below.

Example 3   **Write each decimal as a fraction.**

a. $0.\overline{3}$

Let $N = 0.\overline{3}$ or $0.333\ldots$

Then $10N = 3.\overline{3}$ or $3.333\ldots$

$$\begin{array}{r} 10N = 3.333\ldots \\ -1N = 0.333\ldots \\ \hline 9N = 3 \end{array}$$ Subtract $1N$ from $10N$.

$N = \dfrac{3}{9}$ or $\dfrac{1}{3}$

So, $0.\overline{3} = \dfrac{1}{3}$.

b. $0.\overline{72}$

Let $N = 0.\overline{72}$ or $0.7272\ldots$

Then $100N = 72.7272\ldots$

$$\begin{array}{r} 100N = 72.7272 \\ -1N = 00.7272 \\ \hline 99N = 72 \end{array}$$ Subtract $1N$ from $100N$.

$N = \dfrac{72}{99}$ or $\dfrac{8}{11}$

So, $0.\overline{72} = \dfrac{8}{11}$.

- To write a decimal as a percent, multiply by 100 and add the % symbol. Recall that to multiply by 100, you can move the decimal point two places to the right.

- To write a percent as a decimal, divide by 100 and remove the % symbol. Recall that to divide by 100, you can move the decimal point two places to the left.

Example 4   **Write each decimal as a percent.**

Multiply by 100 and add the % symbol.

a. **0.35**

$0.35 = 0.35$

$\quad = 35\%$

b. **0.06**

$0.06 = 0.06$

$\quad = 6\%$

c. **0.008**

$0.008 = 0.008$

$\quad = 0.8\%$

Example 5   **Write each percent as a decimal.**

Divide by 100 and remove the % symbol.

a. **36%**

$36\% = 36\%$

$\quad = 0.36$

b. **9%**

$9\% = 09\%$

$\quad = 0.09$

c. **120%**

$120\% = 120\%$

$\quad = 1.2$

- To write a fraction as a percent, express the fraction as a decimal. Then express the decimal as a percent.

Example 6 Write each fraction as a percent. Round to the nearest tenth of a percent, if necessary.

a. $\frac{1}{8}$

$\frac{1}{8} = 0.125$

$= 12.5\%$

b. $\frac{2}{3}$

$\frac{2}{3} = 0.6666\ldots$

$= 66.7\%$

c. $\frac{3}{600}$

$\frac{3}{600} = 0.005$

$= 0.5\%$

- To write a percent as a fraction, express the percent as decimal. Then express the decimal as a fraction. Simplify if possible.

Example 7 Write each percent as a fraction.

a. 30%

$30\% = 0.30$

$= \frac{30}{100}$ or $\frac{3}{10}$

b. 140%

$140\% = 1.4$

$= \frac{14}{10}$ or $1\frac{2}{5}$

c. 0.2%

$00.2\% = 0.002$

$= \frac{2}{1000}$ or $\frac{1}{500}$

Exercises Write each fraction as a decimal.

1. $\frac{3}{8}$
2. $\frac{2}{5}$
3. $\frac{2}{3}$
4. $\frac{3}{4}$
5. $\frac{1}{2}$
6. $\frac{5}{9}$
7. $\frac{3}{10}$
8. $\frac{5}{6}$

Write each decimal as a fraction.

9. 0.9
10. 0.25
11. 5.24
12. $0.\overline{45}$
13. $0.\overline{6}$
14. 0.0034
15. 2.08
16. 0.004

Write each decimal as a percent.

17. 0.4
18. 0.08
19. 2.5
20. 0.33
21. 0.065
22. 5
23. 0.005
24. $0.\overline{3}$

Write each percent as a decimal.

25. 45%
26. 3%
27. 68%
28. 115%
29. 200%
30. 0.1%
31. 5.2%
32. 10.5%

Write each fraction as a percent. Round to the nearest tenth of a percent, if necessary.

33. $\frac{3}{4}$
34. $\frac{9}{20}$
35. $\frac{1}{2}$
36. $\frac{1}{6}$
37. $\frac{1}{3}$
38. $\frac{7}{8}$
39. $\frac{6}{5}$
40. $\frac{19}{25}$

Write each percent as a fraction.

41. 70%
42. 3%
43. 52%
44. 25%
45. 6%
46. 135%
47. 0.1%
48. 0.5%

CONTENTS

Making Bar and Line Graphs

- One way to organize data is by using a frequency table. In a **frequency table**, you use **tally marks** to record and display the frequency of events.

Example 1 Make a frequency table to organize the temperature data in the chart at the right.

Noon Temperature (°F)					
52	48	60	39	55	56
60	63	70	58	59	54
63	65	66	73	76	51
54	60	52	48	47	54

Step 1 Make a table with three columns: Temperature, Tally, and Frequency. Add a title.

Step 2 Use intervals to organize the temperatures. In this case, we are using intervals of 10.

Step 3 Use tally marks to record the temperatures in each interval.

Step 4 Count the tally marks in each row and record in the Frequency column.

Noon Temperature (°F)		
Temperature	Tally	Frequency
30–39	I	1
40–49	III	3
50–59	IIII IIII	10
60–69	IIII II	7
70–79	III	3

- A **bar graph** compares different categories of data by showing each as a bar whose length is related to the frequency.

Example 2 The table below shows the results of a survey of students' favorite snacks. Make a bar graph to display the data.

Product	Number of Students
Bagel Chips	10
Fruit	18
Popcorn	15
Potato Chips	20
Pretzels	16
Snack Nuts	9
Tortilla Chips	17

Step 1 Draw a horizontal axis and a vertical axis. Label the axes as shown. Add a title.

Step 2 Draw a bar to represent each category. The vertical scale is the number of students who chose each snack. The horizontal scale identifies the snack chosen.

Favorite Snack Foods
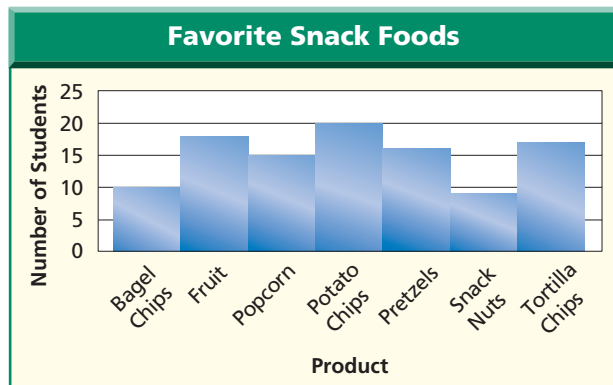
- Another way to represent data is by using a **line graph**. A line graph usually shows how data changes over a period of time.

Example 3

Sales at the Marshall High School Store are shown in the table below. Make a line graph of the data.

School Store Sales Amounts			
September	$670	February	$388
October	$229	March	$412
November	$300	April	$309
December	$168	May	$198
January	$290		

Step 1 Draw a horizontal axis and a vertical axis and label them as shown. Include a title.

Step 2 Plot the points to represent the data.

Step 3 Draw a line connecting each pair of consecutive points.
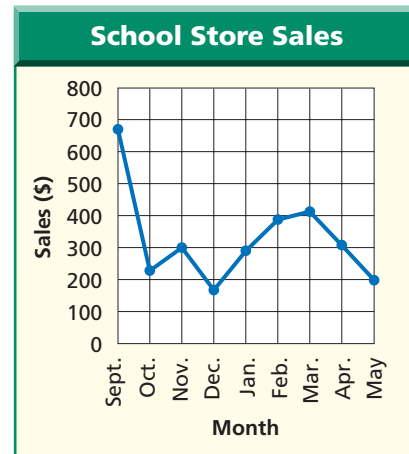
Exercises Determine whether a bar graph or a line graph is the better choice to display the data.

1. the growth of a plant
2. comparison of the populations in Idaho, Montana, and Texas
3. the number of students in each of the classes at your school
4. your height over the past eight years
5. the numbers of your friends that shower in the morning versus the number that shower at night

6. Alana surveyed several students to find the number of hours of sleep they typically get each night. The results are shown at the right. Make a bar graph of the data.

Hours of Sleep					
Alana	8	Kwam	7.5	Tomás	7.75
Nick	8.25	Kate	7.25	Sharla	8.5

7. Marcus started a lawn care service. The chart shows how much money he made over the 15 weeks of summer break. Make a line graph of the data.

Lawn Care Profits ($)								
Week	1	2	3	4	5	6	7	8
Profit	25	40	45	50	75	85	95	95
Week	9	10	11	12	13	14	15	
Profit	125	140	135	150	165	165	175	

8. The frequency table at the right shows the ages of people attending a high school play. Make a bar graph to display the data.

Age	Tally	Frequency
under 20	卌 卌 卌 卌 卌 卌 卌 卌 卌 II	47
20–39	卌 卌 卌 卌 卌 卌 卌 卌 III	43
40–59	卌 卌 卌 卌 卌 卌 I	31
60 and over	卌 III	8

CONTENTS

Making Circle Graphs

A **circle graph** is a graph that shows the relationship between parts of the data and the whole. The circle represents the total data. Individual data are represented by parts of the circle. The examples show how to construct a circle graph.

Example 1 The table shows the percent of her income that Ms. Garcia spends in each category. Make a circle graph to represent the data.

How Ms. Garcia Spends Her Money	
Category	**Amount Spent**
Savings	10%
Car Payment/Insurance	20%
Food	20%
Clothing	10%
Rent	30%
Other	10%

Step 1 Find the number of degrees for each category. Since there are 360° in a circle, multiply each percent by 360 to find the number of degrees for each section of the graph.

Savings, Clothing, Other

$$10\% \text{ of } 360° = 0.1 \cdot 360°$$
$$= 36°$$

The sections for Savings, Clothing, and Other are each 36%.

Car Payment, Food

$$20\% \text{ of } 360° = 0.2 \cdot 360°$$
$$= 72°$$

The sections for Car Payment and Food are each 72°

Rent

$$30\% \text{ of } 360° = 0.3 \cdot 360°$$
$$= 108°$$

The section for Rent is 108°.

Step 2 Use a compass to draw a circle. Then draw a radius.

Step 3 Use a protractor to draw a 36° angle to make the section representing Savings. (You can start with any angle.)

Step 4 Repeat for the remaining sections.

Step 5 Label each section of the graph with the category and percent. Give the graph a title.

How Ms. Garcia
Spends her Money

Savings 10% Car 20% Food 20% Clothing 10% Rent 30% Other 10%

CONTENTS
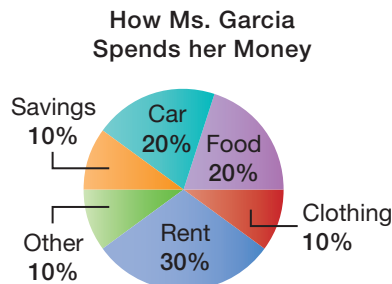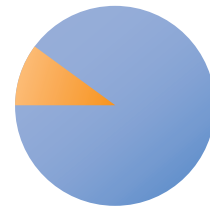
Example 2

The table shows how Jessie uses her time on a typical Saturday. Make a circle graph of the data.

First find the ratio that compares each number of hours to the total number of hours in a day, 24.

Activity	Hours
Jogging	1
Reading	2
Sleeping	9
Eating	2
Talking on the Phone	1
Time with Friends and Family	4
Studying	5

Jogging: $\frac{1}{24}$ Reading: $\frac{2}{24}$ Sleeping: $\frac{9}{24}$ Eating: $\frac{2}{24}$

Phone: $\frac{1}{24}$ Friends: $\frac{4}{24}$ Studying: $\frac{5}{24}$

Then multiply each ratio by 360 to find the number of degrees for each section of the graph.

Jogging, Phone: $\frac{1}{24} \cdot 360° = 15°$

Reading, Eating: $\frac{2}{24} \cdot 360° = 30°$

Sleeping: $\frac{9}{24} \cdot 360° = 135°$

Friends: $\frac{4}{24} \cdot 360° = 60°$

Studying: $\frac{5}{24} \cdot 360° = 75°$

Make the circle graph.

Saturday Time Use
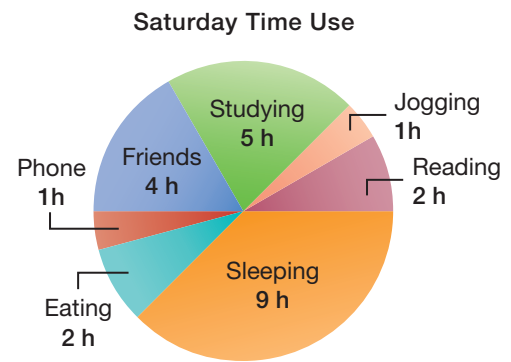
Exercises

1. The table at the right shows the percent of the world's population living in each continent or region. Make a circle graph of the data. (Due to rounding, the percents do not total 100.)

World Population, 2000	
Continent or Region	**Percent of World Total, 2000**
North America	7.9%
South America	5.7%
Europe	12.0%
Asia	60.7%
Africa	13.2%
Australia	0.5%
Antarctica	0%

Source: U.S. Census Bureau

2. The number of bones in each part of the human body is shown in the table at the right. Make a circle graph of the data.

Types of Human Bones	Number
Skull	29
Spine	26
Ribs and Breastbone	25
Shoulders, Arms, and Hands	64
Pelvis, Legs, and Feet	62

CONTENTS

Identifying Two-Dimensional Figures

- Two-dimensional figures can be classified by the number of sides.

Number of Sides	Figure
3	**Tri**angle
4	**Quadri**lateral
5	**Pent**agon
6	**Hex**agon
8	**Oct**agon

The prefixes tell the number of sides.
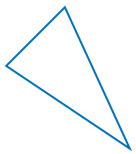
Triangle   Quadrilateral   Pentagon   Hexagon   Octagon
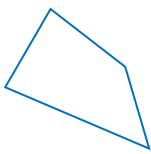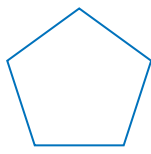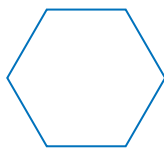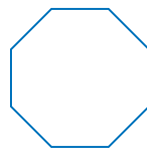
- Triangles can be classified by their angles. An **acute** angle measures less than 90°. An **obtuse** angle measures more than 90°. A **right** angle measures exactly 90°.

Acute Triangle   Obtuse Triangle   Right Triangle
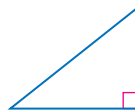
all acute angles   one obtuse angle   one right angle

- Triangles can also be classified by their sides. Recall that **congruent** means having the same measure. Matching marks are used to show congruent parts.

Scalene Triangle   Isosceles Triangle   Equilateral Triangle
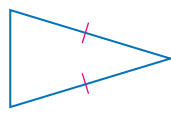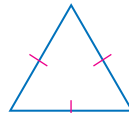
no sides congruent   at least two sides congruent   all sides congruent

Example

Classify each triangle using all names that apply.

a.

b.
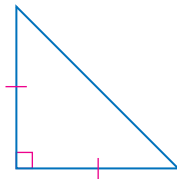
The triangle has one right angle and two congruent sides. It is a right isosceles triangle.

The triangle has one obtuse angle and no congruent sides. It is an obtuse scalene triangle.

CONTENTS

- The diagram below shows how quadrilaterals are classified. Notice that the diagram goes from most general to most specific.

Quadrilateral
polygon with
4 sides

Trapezoid
quadrilateral with one
pair of opposite sides
parallel

Parallelogram
quadrilateral with opposite sides
parallel, opposite sides congruent

Rectangle
parallelogram
with 4 right
angles

Rhombus
parallelogram
with 4 congruent
sides

Square
parallelogram with
4 congruent sides and
4 right angles
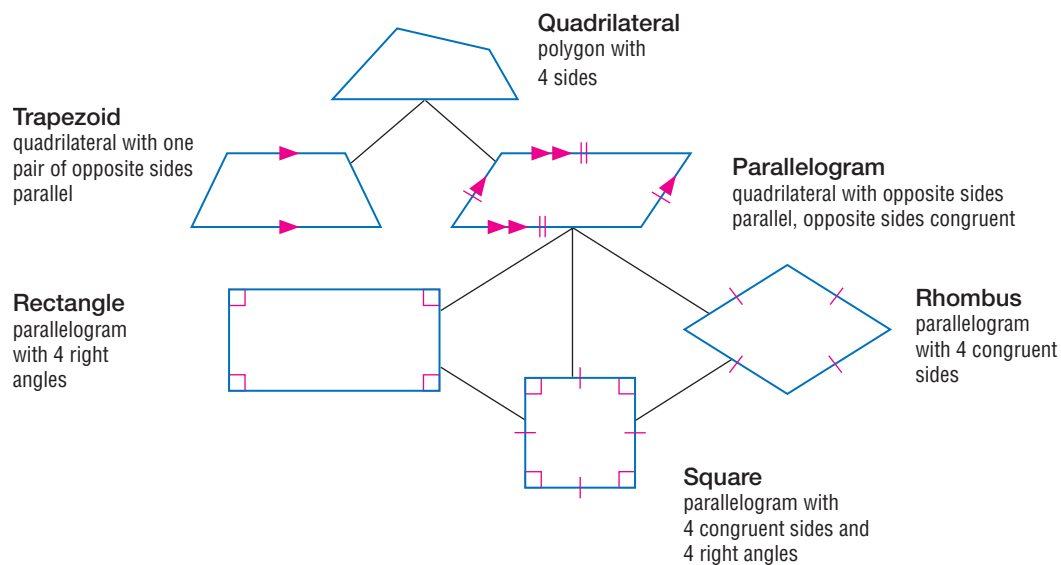
Exercises   **Classify each figure using all names that apply.**

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

12.

13.

14.

15.

Identifying Three-Dimensional Figures

Prisms and pyramids are two types of three-dimensional figures. A **prism** has two parallel, congruent faces called **bases**. A **pyramid** has one base that is a polygon and faces that are triangles.
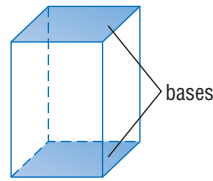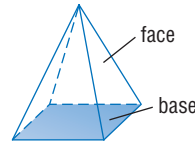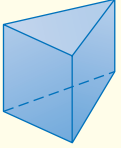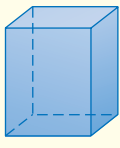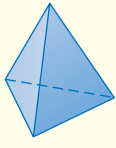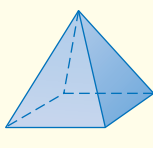
Prism **Pyramid**

Prisms and pyramids are named by the shape of their bases.

Name	triangular prism	rectangular prism	triangular pyramid	rectangular pyramid
Number of Bases	2	2	1	1
Polygon Base	triangle	rectangle	triangle	rectangle
Figure				

A **cube** is a rectangular prism in which all of the faces are squares.
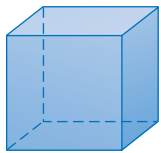
A **cone** is a shape in space that has a circular base and one **vertex**.
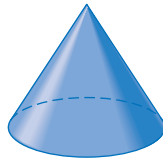
A **sphere** is the set of all points a given distance from a given point called the center.
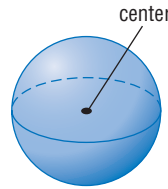
Cube **Cone** **Sphere**

Exercises **Classify each solid figure using the name that *best* describes it.**

1.

2.

3.

4.

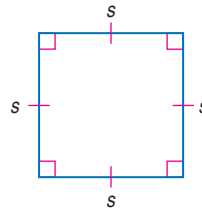5.

6.

7.

8.

9.

CONTENTS
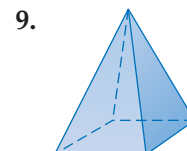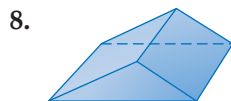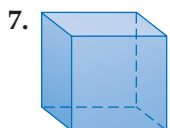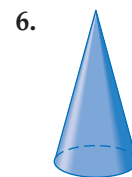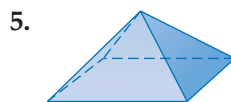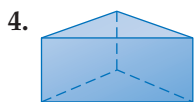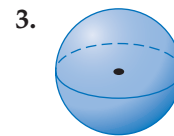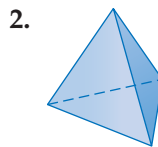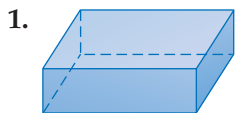
Perimeter and Area of Squares and Rectangles

Perimeter is the distance around a geometric figure. Perimeter is measured in linear units.

- To find the perimeter of a rectangle, multiply two times the sum of the length and width, or $2(\ell + w)$.

- To find the perimeter of a square, multiply four times the length of a side, or $4s$.
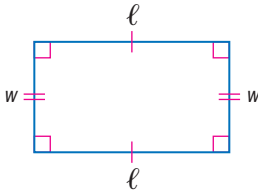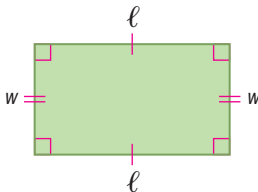
$P = 2(\ell + w)$ or $2\ell + 2w$ $\qquad\qquad\qquad$ $P = 4s$
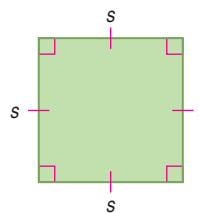
Area is the number of square units needed to cover a surface. Area is measured in square units.

- To find the area of a rectangle, multiply the length times the width, or $\ell \cdot w$.

- To find the area of a square, find the square of the length of a side, or s^2.

$A = \ell w$ $\qquad\qquad\qquad$ $A = s^2$

Example 1 Find the perimeter and area of each rectangle.

a. **A rectangle has a length of 3 units and a width of 5 units.**

$P = 2(\ell + w)$ Perimeter formula
$\quad = 2(3 + 5)$ Replace ℓ with 3 and w with 5.
$\quad = 2(8)$ Add.
$\quad = 16$ Multiply.

$A = \ell \cdot w$ Area formula
$\quad = 3 \cdot 5$ Replace ℓ with 3 and w with 5.
$\quad = 15$ Simplify.

The perimeter is 16 units, and the area is 15 square units.

b. **A rectangle has a length of 1 inch and a width of 10 inches.**

$P = 2(\ell + w)$ Perimeter formula
$\quad = 2(1 + 10)$ Replace ℓ with 1 and w with 10.
$\quad = 2(11)$ Add.
$\quad = 22$ Multiply.

$A = \ell \cdot w$ Area formula
$\quad = 1 \cdot 10$ Replace ℓ with 1 and w with 10.
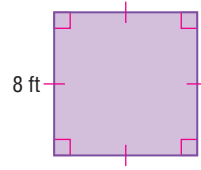$\quad = 10$ Simplify.

The perimeter is 22 inches, and the area is 10 square inches.

CONTENTS

Example 2 Find the perimeter and area of each square.

   a. A square has a side of length 8 feet.

   $P = 4s$ Perimeter formula

       $= 4(8)$ $s = 8$

       $= 32$ Multiply.

   8 ft

   $A = s^2$ Area formula
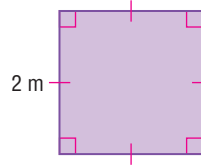
       $= 8^2$ $s = 8$

       $= 64$ Multiply.

   The perimeter is 32 feet, and the area is 64 square feet.

   b. A square has a side of length 2 meters.

   $P = 4s$ Perimeter formula

       $= 4(2)$ $s = 2$

       $= 8$ Multiply.

   2 m

   $A = s^2$ Area formula

       $= 2^2$ $s = 2$
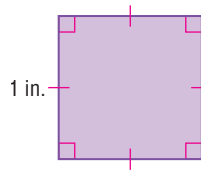
       $= 4$ Multiply.

   The perimeter is 8 meters, and the area is 4 square meters.

Exercises Find the perimeter and area of each figure.

1.

3 cm
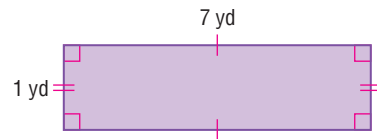
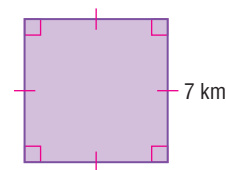2 cm

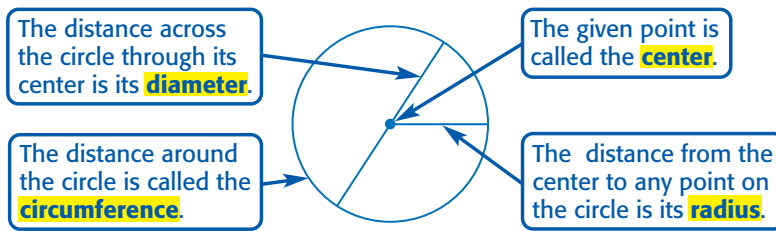**2.**

1 in.

3.

7 yd

1 yd

4.

7 km
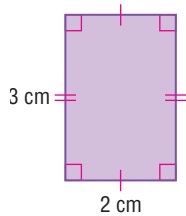
5. a rectangle with length 6 feet and width 4 feet

6. a rectangle with length 12 centimeters and width 9 centimeters

7. a square with length 3 meters

8. a square with length 15 inches

9. a rectangle with width $8\frac{1}{2}$ inches and length 11 inches

10. a rectangular room with width $12\frac{1}{4}$ feet and length $14\frac{1}{2}$ feet

11. a square with length 2.4 centimeters

12. a square garden with length 5.8 meters

13. RECREATION The Granville Parks and Recreation Department uses an empty city lot for a community vegetable garden. Each participant is allotted a space of 18 feet by 90 feet for a garden. What is the perimeter and area of each plot?

CONTENTS

Area and Circumference of Circles

A **circle** is the set of all points in a plane that are the same distance from a given point.

The distance across the circle through its center is its **diameter**.

The given point is called the **center**.

The distance around the circle is called the **circumference**.

The distance from the center to any point on the circle is its **radius**.

- The formula for the circumference of a circle is $C = \pi d$ or $C = 2\pi r$.

Example 1 — **Find the circumference of each circle.**

a. The radius is 3 feet.

Use the formula $C = 2\pi r$.

3 ft

$C = 2\pi r$	Write the formula.
$\quad = 2\pi(3)$	Replace *r* with 3.
$\quad = 6\pi$	Simplify.
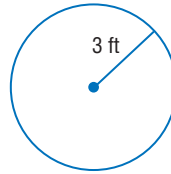
The exact circumference is 6π feet.

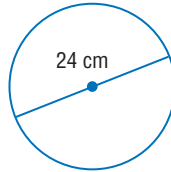6 π ENTER   18.84955592

To the nearest tenth, the circumference is 18.8 feet.

b. The diameter is 24 centimeters.

Use the formula $C = \pi d$.

24 cm

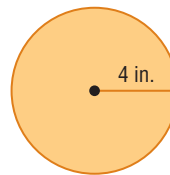|	
$C = \pi d$	Write the formula.
$\quad = \pi(24)$	Replace *d* with 24.
$\quad = 24\pi$	Simplify.
$\quad \approx 75.4$	Use a calculator to evaluate 24π.

The circumference is about 75.4 centimeters.

- The formula for the area of a circle is $A = \pi r^2$.

Example 2 — **Find the area of each circle to the nearest tenth.**

a. The radius is 4 inches.

4 in.

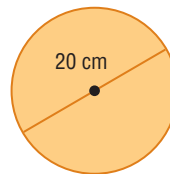|	
$A = \pi r^2$	Write the formula.
$\quad = \pi(4)^2$	Replace *r* with 4.
$\quad = 16\pi$	Simplify.
$\quad \approx 50.3$	Use a calculator to evaluate 16π.

The area of the circle is about 50.3 square inches.

b. The diameter is 20 centimeters.

The radius is one-half times the diameter, or 10 centimeters.

20 cm

$A = \pi r^2$	Write the formula.
$\quad = \pi(10)^2$	Replace *r* with 10.
$\quad = 100\pi$	Simplify.
$\quad \approx 314.2$	Use a calculator to evaluate 100π.

The area of the circle is about 314.2 square centimeters.

CONTENTS

Example 3

HISTORY Stonehenge is an ancient monument in Wiltshire, England. Historians are not sure who erected Stonehenge or why. It may have been used as a calendar. The giant stones of Stonehenge are arranged in a circle 30 meters in diameter. Find the circumference and the area of the circle.

$C = \pi d$ Write the formula.

$\quad = \pi(30)$ Replace d with 30.

$\quad = 30\pi$ Simplify.

$\quad \approx 94.2$ Use a calculator to evaluate 30π.

Find the radius to evaluate the formula for the area. The radius is one-half times the diameter, or 15 meters.

$A = \pi r^2$ Write the formula.

$\quad = \pi(15)^2$ Replace r with 15.

$\quad = 225\pi$ Simplify.

$\quad \approx 706.9$ Use a calculator to evaluate 225π.
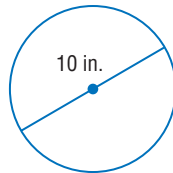
The circumference of Stonehenge is about 94.2 meters, and the area is about 706.9 square meters.

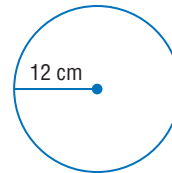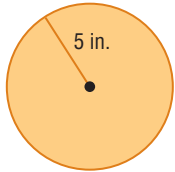**Exercises**   **Find the circumference of each circle. Round to the nearest tenth.**

1.

3 m

2.

10 in.

3.

12 cm
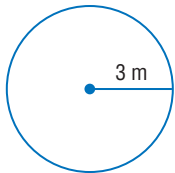
4. The radius is 1.5 kilometers.

5. The diameter is 1 yard.

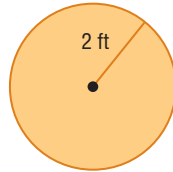**6.** The diameter is $5\frac{1}{4}$ feet.

7. The radius is $24\frac{1}{2}$ inches.

Find the area of each circle. Round to the nearest tenth.

8.

5 in.

9.

2 ft

10.

2 km
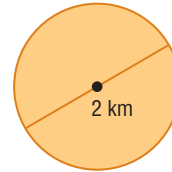
11. The diameter is 4 yards.

12. The radius is 1 meter.

13. The radius is 1.5 feet.

14. The diameter is 15 centimeters.

14. GEOGRAPHY Earth's circumference is approximately 25,000 miles. If you could dig a tunnel to the center of the Earth, how long would the tunnel be?

15. CYCLING The tire for a 10-speed bicycle has a diameter of 27 inches. Find the distance the bicycle will travel in 10 rotations of the tire.

16. PUBLIC SAFETY The Belleville City Council is considering installing a new tornado warning system. The sound emitted from the siren would be heard for a 2-mile radius. Find the area of the region that will benefit from the system.

17. CITY PLANNING The circular region inside the streets at DuPont Circle in Washington, D.C., is 250 feet across. How much area do the grass and sidewalk cover?

CONTENTS

Volume

Volume is the measure of space occupied by a solid. Volume is measured in cubic units. The prism at the right has a volume of 12 cubic units.

$w = 2$
$h = 2$
$\ell = 2$
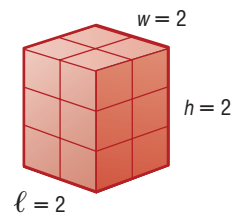
- To find the volume of a rectangular prism, use the formula $V = \ell \cdot w \cdot h$. Stated in words, volume equals length times width times height.

Example

Find the volume of the rectangular prism.

A rectangular prism has a height of 3 feet, width of 4 feet, and length of 2 feet.

2 ft   4 ft   3 ft
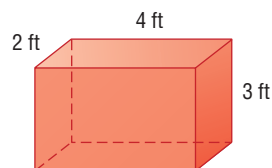
$V = \ell \cdot w \cdot h$ Write the formula.

$V = 2 \cdot 4 \cdot 3$ Replace ℓ with 2, w with 4, and h with 3.
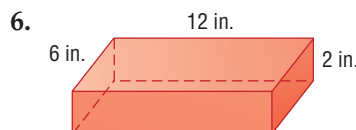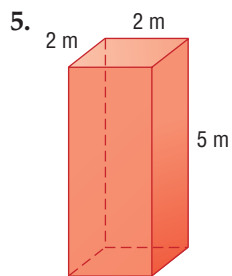
$V = 24$ Simplify.

The volume is 24 cubic feet.

Exercises   Find the volume of each rectangular prism given the length, width, and height.

1. $\ell = 2$ in., $w = 5$ in., $h = \frac{1}{2}$ in.

2. $\ell = 12$ cm, $w = 3$ cm, $h = 2$ cm

3. $\ell = 6$ yd, $w = 2$ yd, $h = 1$ yd

4. $\ell = 100$ m, $w = 1$ m, $h = 10$ m

Find the volume of each rectangular prism.

5.

2 m   2 m   5 m

6.

12 in.   6 in.   2 in.

7. AQUARIUMS   An aquarium is 8 feet long, 5 feet wide, and 5.5 feet deep. What is the volume of the tank?

8. COOKING   What is the volume of a microwave oven that is 18 inches wide by 10 inches long with a depth of $11\frac{1}{2}$ inches?

9. GEOMETRY   A cube measures 2 meters on a side. What is its volume?

FIREWOOD   For Exercises 10–12, use the following.
Firewood is usually sold by a measure known as a cord. A full cord may be a stack $8 \times 4 \times 4$ feet or a stack $8 \times 8 \times 2$ feet.

10. What is the volume of a full cord of firewood?

11. A "short cord" or "face cord" of wood is $8 \times 4 \times$ the length of the logs. What is the volume of a short cord of $2\frac{1}{2}$-foot logs?

12. If you have an area that is 12 feet long and 2 feet wide in which to store your firewood, how high will the stack be if it is a full cord of wood?

CONTENTS

Mean, Median, and Mode

Measures of central tendency are numbers used to represent a set of data. Three types of measures of central tendency are mean, median, and mode.

- The **mean** is the sum of the numbers in a set of data divided by the number of items.

Example 1   **Katherine is running a lemonade stand. She made $3.50 on Tuesday, $4.00 on Wednesday, $5.00 on Thursday, and $4.50 on Friday. What was her mean daily profit?**

$$\text{mean} = \frac{\text{sum of daily profits}}{\text{number of days}}$$

$$= \frac{\$3.50 + \$4.00 + \$5.00 + \$4.50}{4}$$

$$= \frac{\$17.00}{4} \text{ or } \$4.25$$

Katherine's mean daily profit was $4.25.

- The **median** is the middle number in a set of data when the data are arranged in numerical order. If there are an even number of data, the median is the mean of the two middle numbers.

- The **mode** is the number or numbers that appear most often in a set of data. If no item appears most often, the set has no mode.

Example 2   **The table shows the number of hits Marcus made for his team. Find the median of the data.**

To find the median, order the numbers from least to greatest. The median is in the middle.

2, 3, $\underline{3, 5}$, 6, 7

> There is an even number of items. Find the mean of the middle two.

$$\frac{3+5}{2} = 4$$

The median number of hits is 4.

Team Played	Number of Hits by Marcus
Badgers	3
Hornets	6
Bulldogs	5
Vikings	2
Rangers	3
Panthers	7

Example 3   **The table shows the heights of the members of the 2001–2002 University of Kentucky Men's Basketball team. What is the mode of the heights?**

The mode is the number that occurs most frequently. 74 occurs three times, 81 occurs twice, and all the other heights occur once. The mode height is 74.

Player	Height (in.)
Blevins	74
Bogans	77
Camara	83
Daniels	79
Estill	81
Fitch	75
Hawkins	73
Heissenbuttel	76
Parker	80
Prince	81
Sears	78
Smith	74
Stone	82
Tackett	74

Source: ESPN

CONTENTS

- You can use measures of central tendency to solve problems.

Example **4** On her first five history tests, Yoko received the following scores: 82, 96, 92, 83, and 91. What test score must Yoko earn on the sixth test so that her average (mean) for all six tests will be 90%?

$$\text{mean} = \frac{\text{sum of the first five scores} + \text{sixth score}}{6}$$ Write an equation.

$$90 = \frac{82 + 96 + 92 + 83 + 91 + x}{6}$$ Use x to represent the sixth score.

$$90 = \frac{444 + x}{6}$$ Simplify.

$$540 = 444 + x$$ Multiply each side by 6.

$$96 = x$$ Subtract 444 from each side.

To have an average score of 90, Yoko must earn a 96 on the sixth test.

Exercises Find the mean, median, and mode for each set of data.

1. {1, 2, 3, 5, 5, 6, 13}

2. {3, 5, 8, 1, 4, 11, 3}

3. {52, 53, 53, 53, 55, 55, 57}

4. {8, 7, 5, 19}

5. {3, 11, 26, 4, 1}

6. {201, 201, 200, 199, 199}

7. {4, 5, 6, 7, 8}

8. {3, 7, 21, 23, 63, 27, 29, 95, 23}

9. SCHOOL The table shows the cost of some school supplies. Find the mean, median, and mode costs.

Cost of School Supplies	
Supply	**Cost**
Pencils	$0.50
Pens	$2.00
Paper	$2.00
Pocket Folder	$1.25
Calculator	$5.25
Notebook	$3.00
Erasers	$2.50
Markers	$3.50

10. NUTRITION The table shows the number of servings of fruits and vegetables that Cole eats one week. Find the mean, median, and mode.

Cole's Fruits and Vegetable Servings	
Day	**Number of Servings**
Monday	5
Tuesday	7
Wednesday	5
Thursday	4
Friday	3
Saturday	3
Sunday	8

11. TELEVISION RATINGS The ratings for the top television programs during one week are shown in the table at the right. Find the mean, median, and mode of the ratings. Round to the nearest hundredth.

12. EDUCATION Bill's scores on his first four science tests are 86, 90, 84, and 91. What test score must Bill earn on the fifth test so that his average (mean) will be exactly 88?

13. BOWLING Sue's average for 9 games of bowling is 108. What is the lowest score she can receive for the tenth game to have an average of 110?

14. EDUCATION Olivia has an average score of 92 on five French tests. If she earns a score of 96 on the sixth test, what will her new average score be?

Network Primetime Television Ratings	
Program	**Rating**
1	17.6
2	16.0
3	14.1
4	13.7
5	13.5
6	12.9
7	12.3
8	11.6
9	11.4
10	11.4

Source: Nielsen Media Research

CONTENTS